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MAHMOOD NAHVI, PhD • JOSEPH A. EDMINISTER

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Electric Circuits

Seventh Edition

Mahmood Nahvi, PhD

Professor Emeritus of Electrical Engineering
California Polytechnic State University

Joseph A. Edminister

Professor Emeritus of Electrical Engineering
The University of Akron

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Preface

The seventh edition of *Schaum's Outline of Electric Circuits* represents a revision and timely update of materials that expand its scope to the level of similar courses currently taught at the undergraduate level. The new edition expands the information on the frequency response, polar and Bode diagrams, and first- and second-order filters and their implementation by active circuits. Sections on lead and lag networks and filter analysis and design, including approximation method by Butterworth filters, have been added, as have several end-of-chapter problems.

The original goal of the book and the basic approach of the previous editions have been retained. This book is designed for use as a textbook for a first course in circuit analysis or as a supplement to standard texts and can be used by electrical engineering students as well as other engineering and technology students. Emphasis is placed on the basic laws, theorems, and problem-solving techniques that are common to most courses.

The subject matter is divided into 17 chapters covering duly recognized areas of theory and study. The chapters begin with statements of pertinent definitions, principles, and theorems together with illustrative examples. This is followed by sets of supplementary problems. The problems cover multiple levels of difficulty. Some problems focus on fine points and help the student to better apply the basic principles correctly and confidently. The supplementary problems are generally more numerous and give the reader an opportunity to practice problem-solving skills. Answers are provided with each supplementary problem.

The book begins with fundamental definitions, circuit elements including dependent sources, circuit laws and theorems, and analysis techniques such as node voltage and mesh current methods. These theorems and methods are initially applied to DC-resistive circuits and then extended to RLC circuits by the use of impedance and complex frequency. The op amp examples and problems in Chapter 5 have been selected carefully to illustrate simple but practical cases that are of interest and importance to future courses. The subject of waveforms and signals is treated in a separate chapter to increase the student's awareness of commonly used signal models.

Circuit behavior such as the steady state and transient responses to steps, pulses, impulses, and exponential inputs is discussed for first-order circuits in Chapter 7 and then extended to circuits of higher order in Chapter 8, where the concept of complex frequency is introduced. Phasor analysis, sinusoidal steady state, power, power factor, and polyphase circuits are thoroughly covered. Network functions, frequency response, filters, series and parallel resonance, two-port networks, mutual inductance, and transformers are covered in detail. Application of Spice and PSpice in circuit analysis is introduced in Chapter 15. Circuit equations are solved using classical differential equations and the Laplace transform, which permits a convenient comparison. Fourier series and Fourier transforms and their use in circuit analysis are covered in Chapter 17. Finally, two appendixes provide a useful summary of complex number systems and matrices and determinants.

This book is dedicated to our students and students of our students, from whom we have learned to teach well. To a large degree, it is they who have made possible our satisfying and rewarding teaching careers. We also wish to thank our wives, Zahra Nahvi and Nina Edminister, for their continuing support. The contribution of Reza Nahvi in preparing the current edition as well as previous editions is also acknowledged.

MAHMOOD NAHVI
JOSEPH A. EDMINISTER

About the Authors

MAHMOOD NAHVI is professor emeritus of Electrical Engineering at California Polytechnic State University in San Luis Obispo, California. He earned his B.Sc., M.Sc., and Ph.D., all in electrical engineering, and has 50 years of teaching and research in this field. Dr. Nahvi's areas of special interest and expertise include network theory, control theory, communications engineering, signal processing, neural networks, adaptive control and learning in synthetic and living systems, communication and control in the central nervous system, and engineering education. In the area of engineering education, he has developed computer modules for electric circuits, signals, and systems which improve teaching and learning of the fundamentals of electrical engineering. In addition, he is coauthor of *Electromagnetics* in Schaum's Outline Series, and the author of *Signals and Systems* published by McGraw-Hill.

JOSEPH A. EDMINISTER is professor emeritus of Electrical Engineering at the University of Akron in Akron, Ohio, where he also served as assistant dean and acting dean of Engineering. He was a member of the faculty from 1957 until his retirement in 1983. In 1984 he served on the staff of Congressman Dennis Eckart (D-11-OH) on an IEEE Congressional Fellowship. He then joined Cornell University as a patent attorney and later as Director of Corporate Relations for the College of Engineering until his retirement in 1995. He received his B.S.E.E. in 1957 and his M.S.E. in 1960 from the University of Akron. In 1974 he received his J.D., also from Akron. Professor Edminister is a registered Professional Engineer in Ohio, a member of the bar in Ohio, and a registered patent attorney.

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CHAPTER 1

Introduction

1.1 Electrical Quantities and SI Units

The International System of Units (SI) will be used throughout this book. Four basic quantities and their SI units are listed in Table 1-1. The other three basic quantities and corresponding SI units, not shown in the table, are temperature in degrees kelvin (K), amount of substance in moles (mol), and luminous intensity in candelas (cd).

All other units may be derived from the seven basic units. The electrical quantities and their symbols commonly used in electrical circuit analysis are listed in Table 1-2.

Two supplementary quantities are plane angle (also called phase angle in electric circuit analysis) and solid angle. Their corresponding SI units are the radian (rad) and steradian (sr).

Degrees are almost universally used for the phase angles in sinusoidal functions, as in, $\sin(\omega t + 30°)$. (Since ωt is in radians, this is a case of mixed units.)

The decimal multiples or submultiples of SI units should be used whenever possible. The symbols given in Table 1-3 are prefixed to the unit symbols of Tables 1-1 and 1-2. For example, mV is used for millivolt, 10^{-3} V, and MW for megawatt, 10^{6} W.

Table 1-1

Quantity	Symbol	SI Unit	Abbreviation
length	L, l	meter	m
mass	M, m	kilogram	kg
time	T, t	second	s
current	I, i	ampere	A

Table 1-2

Quantity	Symbol	SI Unit	Abbreviation
electric charge	Q, q	coulomb	C
electric potential	V, v	volt	V
resistance	R	ohm	Ω
conductance	G	siemens	S
inductance	L	henry	H
capacitance	C	farad	F
frequency	f	hertz	Hz
force	F, f	newton	N
energy, work	W, w	joule	J
power	P, p	watt	W
magnetic flux	ϕ	weber	Wb
magnetic flux density	$\mathbf{B}$	tesla	T

Table 1-3

Prefix	Factor	Symbol
pico	10^{-12}	p
nano	10^{-9}	n
micro	10^{-6}	μ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
kilo	10^{3}	k
mega	10^{6}	M
giga	10^{9}	G
tera	10^{12}	T

1.2 Force, Work, and Power

The derived units follow the mathematical expressions which relate the quantities. From "force equals mass times acceleration," the *newton* (N) is defined as the unbalanced force that imparts an acceleration of 1 meter per second squared to a 1-kilogram mass. Thus, $1\text{N} = 1\ \text{kg} \cdot \text{m/s}^2$.

Work results when a force acts over a distance. A *joule* of work is equivalent to a *newton-meter*: 1 J = 1 N · m. Work and energy have the same units.

Power is the rate at which work is done or the rate at which energy is changed from one form to another. The unit of power, the *watt* (W), is one joule per second (J/s).

EXAMPLE 1.1 In simple rectilinear motion, a 10-kg mass is given a constant acceleration of 2.0 m/s^2. (*a*) Find the acting force *F*. (*b*) If the body was at rest at $t = 0$, $x = 0$, find the position, kinetic energy, and power for $t = 4$ s.

(*a*) $$F = ma = (10\ \text{kg})(2.0\ \text{m/s}^2) = 20.0\ \text{kg} \cdot \text{m/s}^2 = 20.0\ \text{N}$$

(*b*) At $t = 4$ s, $$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(2.0\,\text{m/s}^2)(4\,\text{s})^2 = 16.0\ \text{m}$$

$$\text{KE} = Fx = (20.0\,\text{N})(16.0\,\text{m}) = 320\ \text{N} \cdot \text{m} = 0.32\ \text{kJ}$$

$$P = \text{KE}/t = 0.32\ \text{kJ}/4\text{s} = 0.08\ \text{kJ/s} = 0.08\ \text{kW}$$

1.3 Electric Charge and Current

The unit of current, the *ampere* (A), is defined as the constant current in two parallel conductors of infinite length and negligible cross section, 1 meter apart in vacuum, which produces a force between the conductors of 2.0×10^{-7} newtons per meter length. A more useful concept, however, is that current results from charges in motion, and 1 ampere is equivalent to 1 coulomb of charge moving across a fixed surface in 1 second. Thus, in time-variable functions, $i(\text{A}) = dq/dt(\text{C/s})$. The derived unit of charge, the *coulomb* (C), is equivalent to an ampere-second.

The moving charges may be positive or negative. Positive ions, moving to the left in a liquid or plasma suggested in Fig. 1-1(*a*), produce a current *i*, also directed to the left. If these ions cross the plane surface *S* at the rate of *one coulomb per second*, then the resulting current is 1 ampere. Negative ions moving to the right as shown in Fig. 1-1(*b*) also produce a current directed to the left.

Of more importance in electric circuit analysis is the current in metallic conductors which takes place through the motion of electrons that occupy the outermost shell of the atomic structure. In copper, for example, one electron in the outermost shell is only loosely bound to the central nucleus and moves freely from one atom to the next in the crystal structure. At normal temperatures there is constant, random motion of these electrons. A reasonably accurate picture of conduction in a copper conductor is that approximately 8.5×10^{28} *conduction* electrons per cubic meter are free to move. The electron charge is $-e = -1.602 \times 10^{-19}$ C,

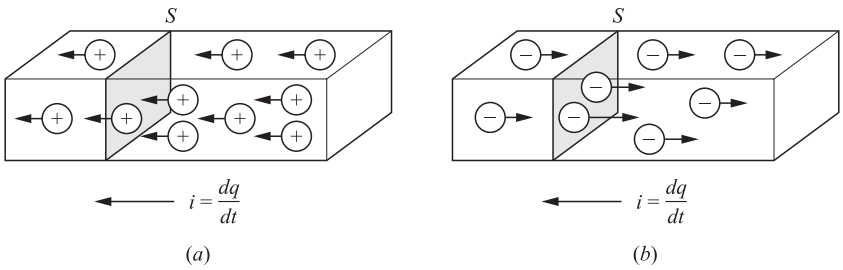


Fig. 1-1

so that for a current of one ampere approximately 6.24×10^{18} electrons per second would have to pass a fixed cross section of the conductor.

EXAMPLE 1.2 A conductor has a constant current of 5 amperes. How many electrons pass a fixed point on the conductor in 1 minute?

$$5\,\text{A} = (5\,\text{C/s})(60\,\text{s/min}) = 300\ \text{C/min}$$

$$\frac{300\,\text{C/min}}{1.602 \times 10^{-19}\,\text{C/electron}} = 1.87 \times 10^{21}\ \text{electrons/min}$$

1.4 Electric Potential

An electric charge experiences a force in an electric field which, if unopposed, will accelerate the charge. Of interest here is the work done to move the charge against the field as suggested in Fig. 1-2(*a*). Thus, if *1 joule* of work is required to move the *1 coulomb* charge Q, from position 0 to position 1, then position 1 is at a potential of *1 volt* with respect to position 0; 1 V = 1 J/C. This electric potential is capable of doing work just as the mass in Fig. 1-2(*b*), which was raised against the gravitational force g to a height h above the ground plane. The potential energy mgh represents an ability to do work when the mass m is released. As the mass falls, it accelerates and this potential energy is converted to kinetic energy.

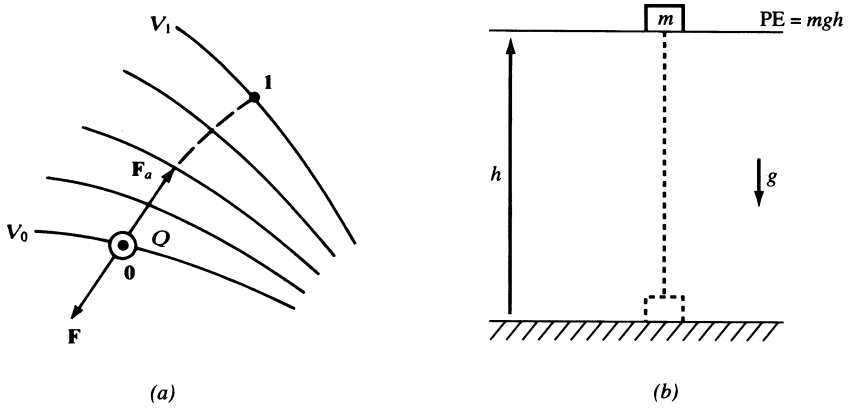


Fig. 1-2

EXAMPLE 1.3 In an electric circuit, an energy of 9.25 μJ is required to transport 0.5 μC from point *a* to point *b*. What electric potential difference exists between the two points?

$$1\ \text{volt} = 1\ \text{joule per coulomb} \qquad V = \frac{9.25 \times 10^{-6}\,\text{J}}{0.5 \times 10^{-6}\,\text{C}} = 18.5\ \text{V}$$

1.5 Energy and Electrical Power

Electric energy in joules will be encountered in later chapters dealing with capacitance and inductance whose respective electric and magnetic fields are capable of storing energy. The rate, in *joules per second*, at which energy is transferred is electric power in *watts*. Furthermore, the product of voltage and current yields the electric power, $p = vi$; 1 W = 1 V · 1 A. Also, V · A = (J/C) · (C/s) = J/s = W. In a more fundamental sense power is the time derivative $p = dw/dt$, so that instantaneous power p is generally a function of time. In the following chapters time average power P_{avg} and a root-mean-square (RMS) value for the case where voltage and current are sinusoidal will be developed.

EXAMPLE 1.4 A resistor has a potential difference of 50.0 V across its terminals and 120.0 C of charge per minute passes a fixed point. Under these conditions at what rate is electric energy converted to heat?

$$(120.0\ \text{C/min})/(60\ \text{s/min}) = 2.0\ \text{A} \qquad P = (2.0\ \text{A})(50.0\ \text{V}) = 100.0\ \text{W}$$

Since 1 W = 1 J/s, the rate of energy conversion is 100 joules per second.

1.6 Constant and Variable Functions

To distinguish between constant and time-varying quantities, capital letters are employed for the constant quantity and lowercase for the variable quantity. For example, a constant current of 10 amperes is written $I = 10.0$ A, while a 10-ampere time-variable current is written $i = 10.0\,f(t)$ A. Examples of common functions in circuit analysis are the sinusoidal function $i = 10.0 \sin \omega t$ (A) and the exponential function $v = 15.0\, e^{-at}$ (V).

SOLVED PROBLEMS

1.1 The force applied to an object moving in the x direction varies according to $F = 12/x^2$ (N). (*a*) Find the work done in the interval 1 m ≤ x ≤ 3 m. (*b*) What constant force acting over the same interval would result in the same work?

(*a*) $$dW = F\,dx \qquad \text{so} \qquad W = \int_1^3 \frac{12}{x^2}\,dx = 12\left[\frac{-1}{x}\right]_1^3 = 8\,\text{J}$$

(*b*) $$8\,\text{J} = F_c(2\,\text{m}) \qquad \text{or} \qquad F_c = 4\ \text{N}$$

1.2 Electrical energy is converted to heat at the rate of 7.56 kJ/min in a resistor which has 270 C/min passing through. What is the voltage difference across the resistor terminals?

From $P = VI$,

$$V = \frac{P}{I} = \frac{7.56 \times 10^3\,\text{J/min}}{270\,\text{C/min}} = 28\,\text{J/C} = 28\ \text{V}$$

1.3 A certain circuit element has a current $i = 2.5 \sin \omega t$ (mA), where ω is the angular frequency in rad/s, and a voltage difference $v = 45 \sin \omega t$ (V) between its terminals. Find the average power P_{avg} and the energy W_T transferred in one period of the sine function.

Energy is the time-integral of instantaneous power:

$$W_T = \int_0^{2\pi/\omega} vi\,dt = 112.5 \int_0^{2\pi/\omega} \sin^2 \omega t\,dt = \frac{112.5\pi}{\omega}(\text{mJ})$$

The average power is then

$$P_{avg} = \frac{W_T}{2\pi/\omega} = 56.25\ \text{mW}$$

Note that P_{avg} is independent of ω.

1.4 The unit of energy commonly used by electric utility companies is the kilowatt-hour (kWh). (*a*) How many joules are in 1 kWh? (*b*) A color television set rated at 75 W is operated from 7:00 p.m. to 11:30 p.m. What total energy does this represent in kilowatt-hours and in mega-joules?

(*a*) 1 kWh = (1000 J/s)(3600 s) = 3.6 MJ
(*b*) (75.0 W)(4.5 h) = 337.5 Wh = 0.3375 kWh
(0.3375 kWh)(3.6 MJ/kWh) = 1.215 MJ

1.5 An AWG #12 copper wire, a size in common use in residential wiring, contains approximately 2.77×10^{23} free electrons per meter length, assuming one free conduction electron per atom. What percentage of these electrons will pass a fixed cross section if the conductor carries a constant current of 25.0 A?

$$\frac{25.0\,\text{C/s}}{1.602 \times 10^{-19}\,\text{C/electron}} = 1.56 \times 10^{20}\,\text{electron/s}$$

$$(1.56 \times 10^{20}\ \text{electron/s})(60\,\text{s/min}) = 9.36 \times 10^{21}\,\text{electrons/min}$$

$$\frac{9.36 \times 10^{21}}{2.77 \times 10^{23}}(100) = 3.38\%$$

1.6 How many electrons pass a fixed point in a 100-watt light bulb in 1 hour if the applied constant voltage is 120 V?

$$100\,\text{W} = (120\ \text{V}) \times I(\text{A}) \qquad I = 5/6\,\text{A}$$

$$\frac{(5/6\,\text{C/s})(3600\,\text{s/h})}{1.602 \times 10^{-19}\,\text{C/electron}} = 1.87 \times 10^{22}\ \text{electrons per hour}$$

1.7 A typical 12 V auto battery is rated according to *ampere-hours*. A 70-A · h battery, for example, at a discharge rate of 3.5 A has a life of 20 h. (*a*) Assuming the voltage remains constant, obtain the energy and power delivered in a complete discharge of the preceding battery. (*b*) Repeat for a discharge rate of 7.0 A.

(*a*) (3.5 A)(12 V) = 42.0 W (or J/s)
(42.0 J/s)(3600 s/h)(20 h) = 3.02 MJ
(*b*) (7.0 A)(12 V) = 84.0 W
(84.0 J/s)(3600 s/h)(10 h) = 3.02 MJ

The ampere-hour rating is a measure of the energy the battery stores; consequently, the energy transferred for total discharge is the same whether it is transferred in 10 hours or 20 hours. Since power is the rate of energy transfer, the power for a 10-hour discharge is twice that in a 20-hour discharge.

SUPPLEMENTARY PROBLEMS

1.8 Obtain the work and power associated with a force of 7.5×10^{-4} N acting over a distance of 2 meters in an elapsed time of 14 seconds. *Ans.* 1.5 mJ, 0.107 mW

1.9 Obtain the work and power required to move a 5.0-kg mass up a frictionless plane inclined at an angle of 30° with the horizontal for a distance of 2.0 m along the plane in a time of 3.5 s. *Ans.* 49.0 J, 14.0 W

1.10 Work equal to 136.0 joules is expended in moving 8.5×10^{18} electrons between two points in an electric circuit. What potential difference does this establish between the two points? *Ans.* 100 V

1.11 A pulse of electricity measures 305 V, 0.15 A, and lasts 500 μs. What power and energy does this represent?

Ans. 45.75 W, 22.9 mJ

1.12 A unit of power used for electric motors is the *horsepower* (hp), equal to 746 watts. How much energy does a 5-hp motor deliver in 2 hours? Express the answer in MJ. *Ans.* 26.9 MJ

1.13 For $t \geq 0$, $q = (4.0 \times 10^{-4})(1 - e^{-250t})$ (C). Obtain the current at $t = 3$ ms. *Ans.* 47.2 mA

1.14 A certain circuit element has the current and voltage

$$i = 10e^{-5000t}\,(\text{A}) \qquad v = 50(1 - e^{-5000t})\,(\text{V})$$

Find the total energy transferred during $t \geq 0$. *Ans.* 50 mJ

1.15 The *capacitance* of a circuit element is defined as Q/V, where Q is the magnitude of charge stored in the element and V is the magnitude of the voltage difference across the element. The SI derived unit of capacitance is the *farad* (F). Express the farad in terms of the basic units. *Ans.* $1\ \text{F} = 1(\text{A}^2 \cdot \text{s}^4)/(\text{kg} \cdot \text{m}^2)$

CHAPTER 2

Circuit Concepts

2.1 Passive and Active Elements

An electrical device is represented by a *circuit diagram* or *network* constructed from series and parallel arrangements of two-terminal elements. The analysis of the circuit diagram predicts the performance of the actual device. A two-terminal element in general form is shown in Fig. 2-1, with a single device represented by the rectangular symbol and two perfectly conducting leads ending at connecting points *A* and *B*. *Active* elements are voltage or current sources which are able to supply energy to the network. Resistors, inductors, and capacitors are *passive* elements which take energy from the sources and either convert it to another form or store it in an electric or magnetic field.

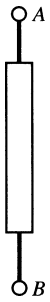


Fig. 2-1

Figure 2-2 illustrates seven basic circuit elements. Elements (*a*) and (*b*) are voltage sources and (*c*) and (*d*) are current sources. A voltage source that is not affected by changes in the connected circuit is an *independent* source, illustrated by the circle in Fig. 2-2(*a*). A *dependent* voltage source which changes in some described manner with the conditions on the connected circuit is shown by the diamond-shaped symbol in Fig. 2-2(*b*). Current sources may also be either independent or dependent and the corresponding symbols are shown in (*c*) and (*d*). The three passive circuit elements are shown in Fig. 2-2(*e*), (*f*), and (*g*).

The circuit diagrams presented here are termed *lumped-parameter* circuits, since a single element in one location is used to represent a distributed resistance, inductance, or capacitance. For example, a coil consisting of a large number of turns of insulated wire has resistance throughout the entire length of the wire. Nevertheless, a single resistance *lumped* at one place as in Fig. 2-3(*b*) or (*c*) represents the distributed resistance. The inductance is likewise lumped at one place, either in series with the resistance as in (*b*) or in parallel as in (*c*).

In general, a coil can be represented by either a series or a parallel arrangement of circuit elements. The frequency of the applied voltage may require that one or the other be used to represent the device.

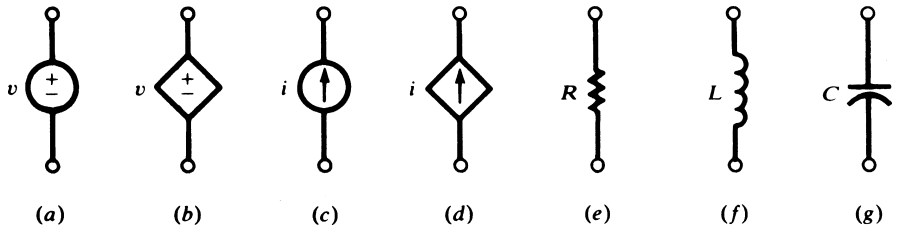


Fig. 2-2

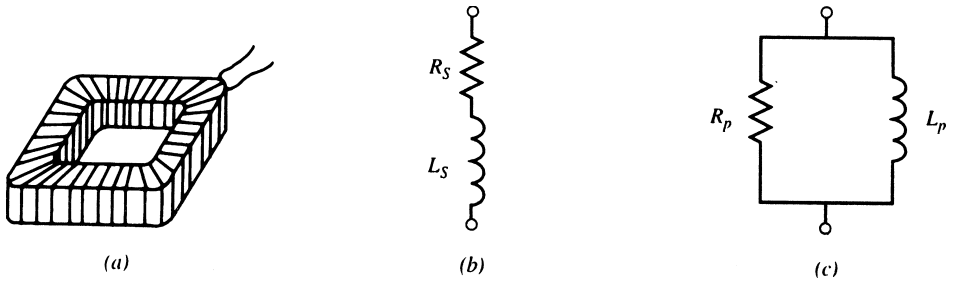


Fig. 2-3

2.2 Sign Conventions

A voltage function and a polarity must be specified to completely describe a voltage source. The polarity marks, + and −, are placed near the conductors of the symbol that identifies the voltage source. If, for example, $v = 10.0 \sin \omega t$ in Fig. 2-4(a), terminal A is positive with respect to B for $0 < \omega t < \pi$, and B is positive with respect to A for $\pi < \omega t < 2\pi$ for the first cycle of the sine function.

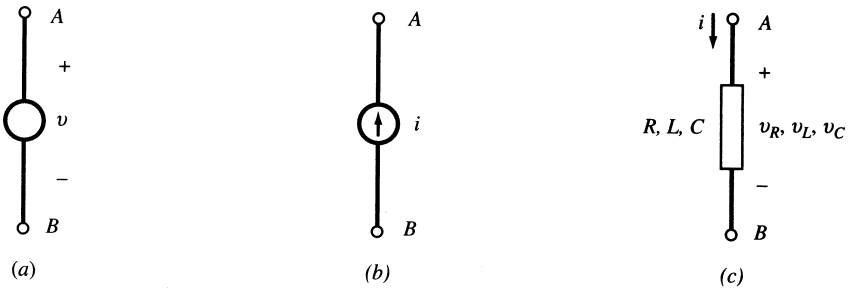


Fig. 2-4

Similarly, a current source requires that a direction be indicated, as well as the function, as shown in Fig. 2-4(b). For passive circuit elements R, L, and C, shown in Fig. 2-4(c), the terminal where the current enters is generally treated as positive with respect to the terminal where the current leaves.

The sign on power is illustrated by the dc circuit of Fig. 2-5(a) with constant voltage sources $V_A = 20.0$ V and $V_B = 5.0$ V and a single 5-Ω resistor. The resulting current of 3.0 A is in the clockwise direction. Considering now Fig. 2-5(b), power is absorbed by an element when the current enters the element at the positive terminal. Power, computed by VI or I^2R, is therefore absorbed by both the resistor and the V_B source, 45.0 W and 15 W, respectively. Since the current enters V_A at the negative terminal, this element is the power source for the circuit. $P = VI = 60.0$ W confirms that the power absorbed by the resistor and the source V_B is provided by the source V_A.

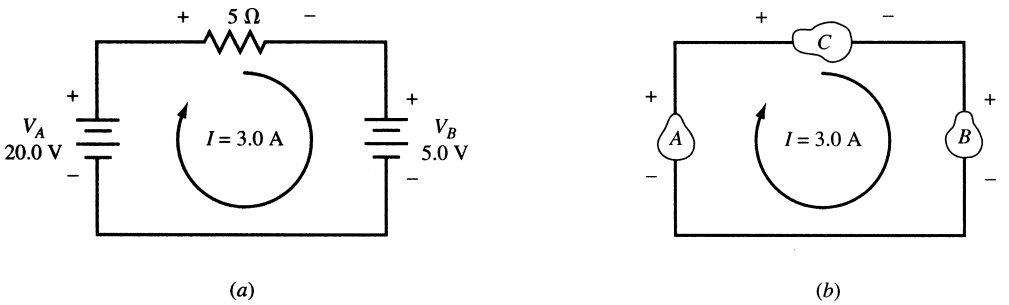
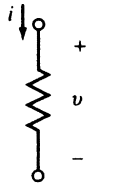
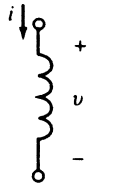
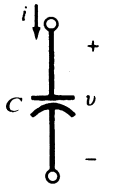


Fig. 2-5

2.3 Voltage-Current Relations

The passive circuit elements resistance R, inductance L, and capacitance C are defined by the manner in which the voltage and current are related for the individual element. For example, if the voltage v and current i for a single element are related by a constant, then the element is a resistance, R is the constant of proportionality, and $v = Ri$. Similarly, if the voltage is proportional to the time derivative of the current, then the element is an inductance, L is the constant of proportionality, and $v = L\,di/dt$. Finally, if the current in the element is proportional to the time derivative of the voltage, then the element is a capacitance, C is the constant of proportionality, and $i = C\,dv/dt$. Table 2-1 summarizes these relationships for the three passive circuit elements. Note the current directions and the corresponding polarity of the voltages.

Table 2-1

Circuit element	Units	Voltage	Current	Power
 **Resistance**	ohms (Ω)	$v = Ri$ **(Ohm's law)**	$i = \frac{v}{R}$	$p = vi = i^2R$
 **Inductance**	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
 **Capacitance**	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$